EVALUATION OF SUPERIMPOSED AND OPTIMAL NETWORKS

A Thesis Submitted
In Partial fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

By
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to the
INDUSTRIAL AND MANAGEMENT ENGINEERING PROGRAMME
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
OCTOBER, 1976

CERTIFICATE

Certified that this work on 'Evaluation of Superimposed and Optimal Networks' by M.S. Cheema has been carried out under our supervision and that this has not been submitted elsewhere for award of a degree.

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ACKNOWLEDGEMENTS

It gives me great pleasure to express my indebtedness to Dr. S.K. Gupta for having initiated me into this topic. I feel elated in manifesting my profound sense of gratitude to Dr. J.L. Batra and Dr. S.K. Gupta for their lively interest, constructive criticism and direction throughout the course of this work.

My sincere thanks are due to those who made my stay at IIT Kanpur a cherishable memory.

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CONTENTS

SYNOPSIS

The increase in demand as time passes is met with by super-imposing new facilities, i.e., by expanding the capacity of the existing facilities. Such a network is called the super-imposed network. Alternatively, it may be assumed that no facility exists and an optimal network is obtained to meet the total demand at that time. Once this network is obtained, it is compared with the existing network. The existing network is modified by retaining, scrapping, expanding and adding of facilities to obtain the optimal network. The evaluation of the alternatives is based on economic considerations. For a given time period t, the alternative which results in minimum total annual cost is selected. The annual cost comprises of fixed, operating and transportation cost. For a facility, the fixed and the operating costs are taken as step function and concave function respectively, whereas the transportation cost is a linear function of the number of units supplied from a facility to a demand center and the distance between them.

For the numerical example considered, the total costs of the super-imposed and the optimal network at time t $\{t = 1,2,\ldots,n\}$, to meet the total demand $\sum_{j \in J} d_{jt}$ are evaluated. The nature of the total cost curves obtained for

he super-imposed and the optimal network are studied. Both he curves are initially concave increasing and later the ptimal (super-imposed) total cost curve is concave increasing convex increasing). It is believed that the nature of the otal cost curves shall not vary for various demand patterns.

CHAPTER 1

INTRODUCTION

The increase in demand for most of the commodities depends upon many factors, population growth being the major contributing factor. Also, in general, it can be assumed that for certain commodities like electricity, water supply, transportation, etc., the demand to be met shall increase in the foreseeable future. New industrial units may also be set-up at undeveloped places (as a policy), as a result of which the demand potential at new centers is also to be taken into consideration. Some resources may become available in future to be tapped at new places resulting in enlarging the number of locations to be considered.

Due to the economies-of-operation and the technological considerations, only short range planning is normally done. Any increase in demand, as time passes, is met with by superimposing new facilities or by expanding the capacity of the existing facilities.

A somewhat similar situation to the problem considered in this thesis is the conversion of meter gauge railway track to the broad gauge track throughout the country, considering the economies of operation. Initially, a meter gauge track

was laid between Kandla and Delhi to meet the needs of the time. But now Kandla has become a major sea-port, handling most of the imports. At Udaipur, due to mining of rock phosphates and other industries the movement of goods has increased considerably. Thus railways have decided to undertake a project to convert meter gauge line between Kandla-Udaipur-Ahmedabad-Delhi to broad gauge.

Another situation which warrants such considerations concerns the evolution of UPSRTC Kanpur region workshop depots due to the increase in the size of the fleet. For better management, efficient use of services, equipments, and manpower, it shall be worth consideration of doing away with the practices in vogue and to build a large central workshop at a new place and to decentralize a few operations which are at present carried over at the central workshop.

Let us assume that during an interval of time 't' from t_1 to t_2, the demand increases from x_1 to x_2 units. Now, if we try to build a network, assuming no facility exists, to supply x_2 units at time t_2 in such a manner as to minimize the total cost (the sum of the fixed operating and transportation costs), the network so obtained shall cost less than or equal to the super-imposed network at time t_2.

Consider the cost structure for changing over from the super-imposed network of time t_1 to some other network of time t_2. Concepts of engineering economics are used to estimate the relizable value of the facilities in existence, which are to be scrapped as these donot form part of the network of time t_2. The difference in the present worth and the relizable value is the loss incurred by scrapping a facility. This amount is to be added to the total cost of the new network. The total cost of such a network may be more than the total cost of the super-imposed network, and under such circumstances it shall be advisable to prefer the super-imposed network to meet the demand. But it is envisaged that a time may come when the increase in demand shall be sufficient enough to warrant the switching over from the super-imposed to a new network. It may be stressed here that there are certain other considerations, socio-political for example, which can go against such a changeover.

In between the above mentioned two extreme cases, there exists a distinct possibility of obtaining more economical networks. This kind of a possibility arises when the increase in demand is not sufficient enough to warrant change-over from the evolved to the optimal network. In such a case, instead of scrapping all the facilities which do not form part of the optimal network, only some facilities are

scrapped while the other facilities are retained. The network so obtained shall operate at a total cost which is lower than the total cost of both i.e. the super-imposed and the optimal network.

This thesis, however, is concerned with the study of only two distinct types of networks, namely, the super-imposed and the optimal networks. Taking into considerations only the economic aspects, an attempt is made to study such a problem.

The following methods need to be evolved to study the problem posed above:

(a) a method to find out the super-imposed network and to calculate its total cost.

(b) a method to determine the optimal network to meet the total demand, disregarding the existing facilities.

(c) a method to calculate the costs involved in changing over from a given network to some other given network, where both the networks meet the same demand.

THESIS CONTENTS:

In addition to this introductory chapter, there are three more chapters. In Chapter 2, a brief survey of the existing literature relevant to the kind of the problems with which this thesis is concerned, is given.

Chapter 3 is divided into four parts. In part I, the problem is formulated. In part II, a heuristic algorithm following Drysdele and Sandifords(3) approach, is given to solve the problem. In part III, the economic aspects concerning cost considerations are given. The last part gives the methodology to compare the costs. A computer program has been developed for the heuristic algorithm. Appendix gives the listing of the program.

In the last chapter, an example is considered. The computer program developed for the heuristic algorithm is used to find out the optimal and the super-imposed networks to meet the demand. The total costs for the example are computed and the results obtained from the input variables (data) are discussed, with a view to adopt the optimal network as and when the need arises.

CHAPTER 2

LITERATURE SURVEY

The problem of physical distribution of a product is to determine the optimum number of facilities, their locations (chosen from a given set of potential sites), and the quantity supplied to the set of demand centers from each facility in order to minimize the total cost involved. It is a non-convex programming problem. The non-convexities being caused by the economies-of-scale associated with the cost of operating the facilities. These complexities in the cost structure can be represented mathematically by a Non-Linear Programming (NLP) model only. However, it is possible to solve this NLP model approximately using LP approach. The problem may also be approached by constructing a detailed simulation model of the distribution system. The various approaches relevant to this problem are surveyed in this chapter.

2.1 FACILITY LOCATION PROBLEM - NLP MODEL:

In its most general form, the facility location problem can be stated mathematically as follows:

$$\text{Minimize} \quad Z(t) = \sum_{i \in I} [f_i(x_{it}) + g_i(x_{it})] + \sum_{i \in I} \sum_{j \in J} h_{ij}(x_{ijt})$$

Subject to:

$$\sum_i x_{ijt} = d_{jt} \qquad \forall j \in J$$

$$\sum_j x_{ijt} = x_{it} \qquad \forall i \in I$$

$$x_{it}, x_{ijt} \geq 0$$

where, I set of all possible locations under considerations for facility location

J set of all demand centers,

x_{it} number of units produced by facility i at time t,

x_{ijt} number of units supplied from facility i to the demand center j at time t,

d_{jt} demand at j-th demand center, $j = 1, 2, \ldots J$, at time t.

$f_i(x_{it})$ fixed cost at facility i, for producing x_{it} units,

$g_i(x_{it})$ operating costing at facility i, for producing x_{it} units,

$h_{ij}(x_{ijt})$ transportation cost for supplying x_{ijt} units from facility i to the demand center j.

The most important types of non-linearities generally encountered stem from the fixed cost associated with the operation of the facilities, and variable operating cost which are non-linear. Two types of functions for fixed and operating cost have been suggested in literature, strictly concave functions and piecewise linear functions.

2.2 LP APPROACH:

The various approaches which are used to solve this NLP using LP approach are: (a) Baumol-Wolfe Marginal Cost Approach (2), (b) Balinski-Mills Average Cost Approach (1), and (c) Boolean Approach (4).

All these approaches require the problem to be cast in the frame-work of the LP transportation model. Whereas Baumol and Wolfe approximate the concave cost function by a straight line, Balinski and Mill treat the costs as piece-wise linear function.

In the Boolean approach, the capacity constraint at each facility location is taken care of by replacing the facilities by the corresponding pseudo plants. The solution to the mixed integer formulation is obtained by solving a set of Pseudo-Boolean Inequalities (SPBI). Such an approach is developed by Elshafei (4), based on the earlier work of Hammer and Rudeau (6).

2.3 SIMULATION APPROACH:

Simulation is a numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical and logical models that describe the behaviour of business or economic system (or some component thereof) over extended periods of real time. The use of simulation technique in the modeling of facility networks has been proposed by Shycon and Maffei (10), as a means of avoiding the approximations as required by the techniques outlined above. Once a model has been constructed containing the basic characteristics of the distribution system, it is used to evaluate the distribution cost associated with alternative set of facilities, and customer demand. However, the algorithms devised so far, are not able to generate near optimal network.

2.4 HEURISTIC APPROACHES:

In heuristic approaches an attempt is made to obtain a good solution using a comparitively easier analysis.

Heuristic approaches for the facility location problem have been developed by Kuehn and Hamburger (8), Manne (9), Feldmen, Lehrer and Ray (5) and, Drysdale and Sandiford (3). These approaches provide algorithms which are computationally superior and permit the considerations of a much wider class of cost functions, but lack the ability to reach with certainty the precise optimum.

Kuehn and Hamburger (8), have compared their approach with the published efforts of solving the problem either by means of simulation or as a variant of LP.

Drysdale and Sandiford (3), use the approach which is quite similar to that of Kuehn and Hamburger, and Feldman, Lehrer and Ray. It differs from both, however, in the use of two different heuristics simultaneously, namely, the dropping heuristic of Feldman et al, combined with a new heuristic involving stepwise incrementing the fixed cost from zero to its final value. At each terminal the uneconomical supply centres are dropped. The approach includes some systematic local searching at each stage similar to the 'bump and shift' routine of Kuehn and Hamburger. The method is based on the approach developed by Law (7). It uses recursively the unrestricted transportation method for solving the distribution problem.

2.5 MOTIVATION OF THE PRESENT WORK:

The problem as posed in this thesis, concerning the economic consideration of two distinct types of networks namely, the super-imposed and the optimal, with the growth in demand, has not been handled in the past. The literature surveyed above gives only the various techniques to find out the optimal network for a specified demand pattern. A modified version of the Drysdale-Sandiford approach is used to

determine the distribution network to meet the total demand at time t , or to meet the increase in demand between t -1, t.. Some of the well known relationships of engineering economics have been used to evaluate the cost of obtaining the networks of time t from the existing network of time t -1.

CHAPTER 3

PROBLEM FORMULATION AND SOLUTION METHODOLOGY

3.1 FACILITY LOCATION PROBLEM:

Consider at time t, the facilities are to be chosen out of set of locations I, to meet demand d_{jt} at demand center j ($j \in J$), in such a manner as to minimise the total cost (the fixed, operating and transportation cost). This problem is a particular case of the problem stated in Chapter 2, page 7 . The cost functions are assumed to be of the the form:

$$f_i(x_{it}) = \text{a step function}$$

$$g_i(x_{it}) = k_1\, x_{it} - k_2\, x_{it}^2$$

$$h_{ij}(x_{ijt}) = c\, y_{ij}\, x_{ijt}$$

where,

$f_i(x_{it})$	Annual fixed cost of facility i producing the units
k_1, k_2	Constants of the variable operating cost
c	transportation cost/unit distance/unit quantity
y_{ij}	distance between facility i and demand center j.

We assume that the fixed cost, operating cost and transportation cost do not depend on time.

The problem thus becomes:

$$\text{Minimize} \quad Z(t) = \sum_{i \varepsilon I} f_i(x_{it}) + \sum_{i \varepsilon I} (k_1 x_{it} - k_2 x_{it}{}^2)$$

$$+ \sum_{i \varepsilon I} \sum_{j \varepsilon J} c . y_{ij} . x_{jt}$$

$$\text{Subject to} \qquad \sum_{i} x_{ijt} = d_{jt} \qquad \forall\ j \varepsilon J$$

$$\sum_{j} x_{ijt} = x_{it} \qquad \forall\ i \varepsilon I$$

where

$$f_i(x_{it}) = \begin{cases} 0 & \text{for } x_{it} = 0 \\ f_i{}^l & x_{it,l-1} < x_{it} \leq x_{it,l} \quad (l = 1, 2, ..n) \\ \infty & x_{it} > x_{it,n} \end{cases}$$

$$x_{it}, x_{ijt} \geq 0$$

The problem stated above can be solved by using a modified version of the algorithm suggested by Drysdale and Sandiford (3). The algorithm determines, heuristically, the optimum number of facilities, their size and location, the distribution patterns, and the total cost (fixed, operating and transportation) with a view to minimize the total cost subject to that the demand at time t (or, the increase in demand between time t, t + 1) is to be met. The modified algorithm is given below.

ALGORITHM FOR DETERMINING OPTIMAL NETWORK:

Step 1: Find out the transportation cost matrix. $t_{ij} = c \times y_{ij}$.

Step 2: Find out the solution, considering only the transportation cost. This is called the indifferent solution.

Step 3: Take the initial fixed cost $f_i^{'1}$ to be a low value of the final fixed cost f_i^1. (Drysdale and Sandiford have found that a good solution can be obtained if the initial fixed cost is taken equal to 5-10 percent of the final fixed cost and it is incremented in Steps of 5-10 percent). Now, the combined unit transportation and fixed cost is given by,

$$c_{ij} = t_{ij} + f_i^{'1}/x_i$$

Find out the solution as obtained in Step 2, but considering the combined cost c_{ij} instead of only the transportation cost t_{ij}. The reason for considering a fraction of the final fixed cost instead of the final fixed is that the overall process being examined is basically one of eliminating the facilities. When the fixed and operating costs increase sufficiently, the cost of operating the system can be improved by reducing the number of facilities and paying more in transportation charges. To do this in

one step may result in the incorrect elimination of some economical facilities. To avoid this, the fixed cost f_i^{1} is started at a small percentage of its correct value and increased slightly at each iteration until the desired level is reached.

The facilities which donot supply are dropped from further consideration by giving the transportation cost (t_{ij}) for these facilities to the demand centers a very large value.

Step 4: TEST FOR RETENTION OF FACILITIES;

A facility should stay in the network provided it saves in transportation and operating cost more than the fixed cost of the facility. In short,

$$\sum_{j\varepsilon J} (t_{ij}-t_{rj})\, x_{rjt} + k_1(x_{it} + x_{rt}) - k_2(x_{it} + x_{rt})^2$$

$$\geq f_r'^{1} + (k_1\, x_{rt} - k_2\, x_{rt}^{2}) + (k_1\, x_{it} - k_2\, x_{it}^{2})$$

where location i has a facility that is assumed to be justified and location r has a facility being tested.

The set of facilities obtainable step 3 are tested in the following manner.

The first facility in the set is assumed to be justified and all the other facilities in the set

are tested against it. The tested facilities which fail the test are excluded for further consideration. One of the facilities (not already considered justified) out of the set of facilities now under consideration is assumed justified and all the other facilities in the set are tested against it. The process is repeated till all the facilities of the set have been considered justified once. Let us assume that the distribution network obtained in this step contains K facilities.

Step 5: Calculate the total cost of the distribution network obtained in Step 4.

The total cost,

$$T_1 = \sum_{i \epsilon K} \sum_{j \epsilon J} t_{ij} \cdot x_{ijt} + \sum_{i \epsilon K} f_i'^{i} + \sum_{i \epsilon K} (k_1 x_{it} - k_2 x_{it}^2)$$

Step 6: ENSURING NO PROFITABLE FACILITY IS ELIMINATED:

Increment $f_i'^{i}$ by Δf_i^{i} and repeat steps 3 through 5. Let us assume that now there are L facilities in the distribution network and T_2 is the total cost, where $L \leq K$. To make the second solution acceptable, the following condition must hold good:

$$T_2 - T_1 < \sum_{i \epsilon K} \Delta f_i^{i}$$

If the above condition is not satisfied, the better solution obviously corresponds to the distribution network which resulted in the total cost T_1. Let $T_2{}'$ represent the total cost at the current iteration. Then,

$$T_2{}' = T_1 + \sum_{i \varepsilon K} \Delta f_i^{t}$$

If the said condition is satisfied, then,

$$T_2{}' = T_2$$

Step 7: If the fixed facility cost ($f_i^{\prime t}$) is not equal to the final fixed facility cost (f_i^{t}), increment it by Δf_i^{t} and go to Step 3 and repeat Steps 3 through 6. Otherwise, stop.

The network so obtained to meet the total demand at time t is called the optimal network. This network is compared with the existing network. The facilities common to both the networks are retained while the other are scrapped. The capacities of the retained facilities are augumented to the desired levels and new facilities constructed to achieve the optimal network. The present worth (PW) and the net relizable value (NRV) are evaluated for the facilities which have to be scrapped out of the existing network for implementation consideration of the proposed optimal network. The difference

between the PW and the NRV is the loss incurred by scrapping that facility. This loss is appropriately accounted to determine the total annual cost of the optimal network.

DEVELOPMENT OF SUPER-IMPOSED NETWORK:

For the super-imposed network of time t+1, the increase in demand between t, t+1 is supplied in the manner determined by the algorithm given in the previous section. This network is super-imposed on the existing network of t.

3.2 EVALUATION OF OPTIMAL AND SUPER-IMPOSED NETWORKS:

The comparison of these two networks, namely, optimal and super-imposed networks at any time t is carried out on the basis of the total annual cost. The total annual cost of operating a particular network is obtained by summing up the iso-costs corresponding to the fixed cost, the operating cost and the cost of transportation.

To evaluate the annual fixed cost, the present worth and the NRV of a facility, the following relationships from engineering economics have been used.

The iso-cost, R, for the investment is given by,

$$R = (P-L)_{i,n} \; crf + Li$$

where,

P the investment on the equipment; the total first cost; the installed cost,

L the salvage value at the end of the economic life

i the minimum required rate of return,

n the economic life in years on the basis that the rate of return i is for a one year period,

crf 'Capital-Recovery-Factor'.

The present worth PW of a facility after it has operated for m years is obtained by the relationship,

$$PW = R_{1,(n-m)} \text{ uspwf}$$

where,

uspwf 'Uniform-Series-Present-Worth-Factor'.

When a facility is scrapped, its NRV is always less than the PW of the facility. This is because, NRV also accounts for the cost of dismantling, handling, remounting, etc.

Keeping the above relationships in view, the evaluation of the super-imposed and the optimal networks for demand growth at each stage is done in the following manner.

Let at time t and t-1, (when t = 1,2,3,...n), the total demand be d_t and d_{t-1} respectively (where $d_1 < d_2 < \ldots < d_n$). The increase in demand $\Delta d(t, t-1) = d_t - d_{t-1}$.

The total cost $T_{t,0}$ of the optimal network at time t is the sum of the fixed cost $F_{t,0}$ and the variable cost $V_{t,0}$. Let the total fixed cost needed to obtain the optimal network from th

existing network of time t-1 be $F'_{t,0}$. $F'_{t,0}$ includes the fixed cost of the portion of the existing network at the desired capacity, fixed cost for the new facilities added and the losses incurred due to the scrapping of the facilities which do not form part of the optimal network. Thus the total cost of operating the optimal network at time t, $T'_{t,0} = F'_{t,0} + V_{t,0}$.

The total cost $T_{t,s}$, of the super-imposed network at time t is the sum of the total cost of the existing network of time t-1, and the total cost required to meet the increase in demand $\Delta d(t, t-1)$. That is,

$$T_{t,s} = T_{t-1,s} + T_{t,s}(t,t-1)$$

Comparing the total costs for the networks at time t,

If $T'_{t,0} \geq T_{t,s}$, it is economical to have the super-imposed network,

and if $T'_{t,0} < T_{t,s}$, it is economical to switch-over to the optimal network from the super-imposed network of time t-1.

CHAPTER 4

AN EXAMPLE - RESULTS AND DISCUSSIONS

4.1 AN EXAMPLE:

A numerical example is considered in this chapter to illustrate the methodology developed in Chapter 3.

Conforming to the notations of Chapter 3, the input variables are:

$$I = \{1, 2, \ldots, 14\}$$

$$J = \{1, 2, \ldots, 20\}$$

$$c = 0.2$$

$$k_1 = 2.5$$

$$k_2 = 0.0008$$

$$f_1(x_i) = \begin{cases} 0 & \text{for} \quad x_1 = 0 \\ 100 & 0 < x_1 \leq 300 \\ 150 & 300 < x_1 \leq 600 \\ 200 & 600 < x_i \leq 1000 \\ 250 & 1000 < x_i \leq 1500 \\ \infty & x_1 > 1500 \end{cases}$$

n = 20 years

r = 10 percent

NRV = 70 percent of the present worth of the facility.

the distance matrix, y_{ij} for all i and j is given in the Table 1.

the demand matrix, d_{jt} $(j \varepsilon J)$ and $t\{t = 1,2,\ldots,11\}$ is given in Table 2.

the difference between any two consecutive time periods, t and t+1, is 2 years.

It is assumed that the network needed to meet demand $\sum_j d_{jt}$ at time t+1 is available at time t. Lead time to build this network is assumed zero.

4.2 RESULTS:

The results obtained for the numerical example are given in Table 3. The total costs $T'_{t,0}$ and $T_{t,s}$ for the optimal and the super-imposed network at time t (t = 1,2,..,10) are plotted in Figure 1. The curve for the optimal network (super-imposed network) is more or less concave increasing (convex increasing).

4.3 DISCUSSION OF RESULTS:

(a) From Figure 1, it is apparent that the nature of the total cost curve for the optimal network is concave increasing, but however, it is convex increasing between time t = 4,5 and 6. Referring to Table 1, the possible explanations are:

Table 1: Distance Matrix (y_{ij})

Supply Center i	Demand Center j																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2	8	7	12	4	10	12	9	20	18	12	14	3	15	22	12	26	9	17	12
2	6	3	9	8	11	6	8	15	18	25	4	8	11	13	18	5	17	20	20	15
3	9	10	3	17	14	22	7	7	10	22	16	9	7	18	13	14	30	14	9	20
4	18	9	17	1	22	20	8	25	12	30	15	7	23	66	12	8	13	30	25	25
5	4	5	5	11	9	12	6	14	16	22	9	6	6	15	15	8	18	15	14	17
6	16	11	11	6	20	20	4	22	7	31	17	4	17	5	7	11	18	27	15	25
7	11	9	4	11	15	18	3	13	5	24	17	5	13	11	9	14	25	16	7	22
8	10	13	11	26	9	16	18	2	22	12	25	20	8	28	24	20	40	5	20	17
9	16	13	8	14	20	24	6	20	3	29	25	7	16	14	6	18	34	22	5	30
10	15	23	21	31	11	15	24	16	30	4	25	26	15	40	32	24	40	10	30	12
11	13	12	18	16	12	2	20	24	24	24	5	20	17	21	26	10	10	22	30	9
12	11	4	9	5	15	13	5	18	9	25	12	13	14	9	13	7	24	22	16	19
13	7	12	6	20	9	14	12	7	14	18	16	15	2	20	18	15	29	30	13	15
14	8	11	15	20	5	5	19	17	25	11	8	19	20	26	27	13	24	13	27	6

Table 2: Demand Matrix d_{jt}.

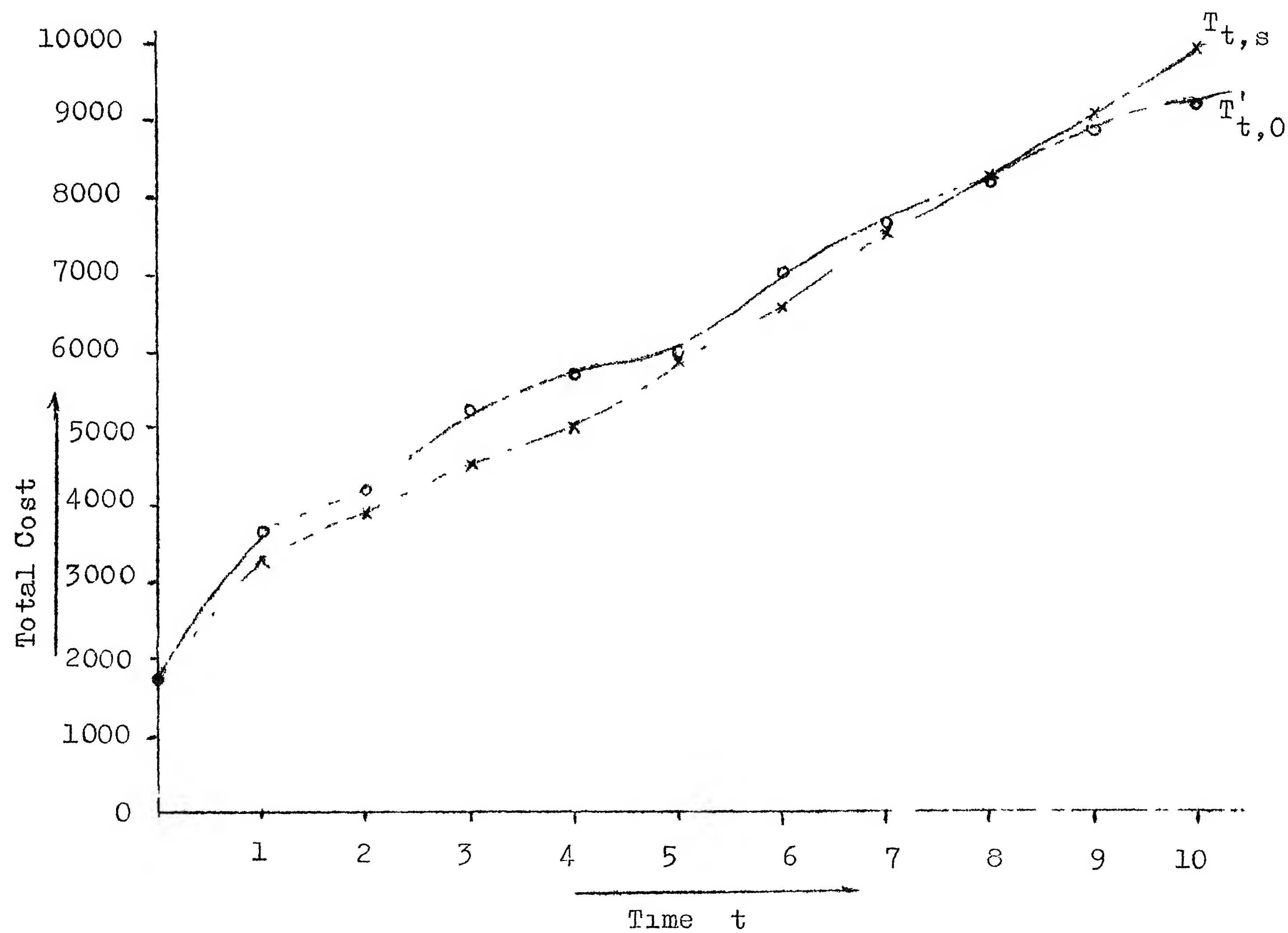
Demand at j	Time t: 1	2	3	4	5	6	7	8	9	10	11
1	0	40	50	65	75	85	100	110	115	120	125
2	0	0	0	0	0	20	25	35	45	55	65
3	0	20	20	25	25	30	35	35	35	40	40
4	0	50	60	70	75	90	100	105	115	120	125
5	0	50	60	80	90	95	110	120	125	135	135
6	0	0	0	20	25	30	35	40	45	45	50
7	0	40	60	69	75	80	90	100	110	115	120
8	0	0	0	0	0	0	10	20	35	50	70
9	120	150	165	175	180	190	200	210	215	225	235
10	0	0	0	0	0	0	10	20	40	60	75
11	0	0	0	0	0	25	30	35	45	55	65
12	120	130	140	150	160	165	175	135	195	205	215
13	0	30	45	60	65	70	85	95	100	105	110
14	120	140	155	170	180	190	200	210	220	230	240
15	30	70	85	90	95	110	115	120	130	135	140
16	0	0	0	0	0	0	20	25	35	45	55
17	0	10	10	15	15	20	25	25	30	30	30
18	0	0	0	0	0	0	15	30	50	70	85
19	150	170	180	190	200	210	215	225	235	250	255
20	0	0	0	20	25	30	30	35	40	40	40
Total Demand	540	900	1030	1195	1285	1440	1620	1780	1960	2130	2275

Table 3: Total Costs of Super-Imposed and Optimal Networks at Time t.

Network	OPTIMAL					SUPER-IMPOSED			
Time t	I	$F_{t,O}$	$V_{t,O}$	$F'_{t,O}$	$T'_{t,O}$	I	$T_{t,s}$	$T_{t-1,s}$	$T_{t,s}$
0	-	-	-	-	-	6,9+	-	-	1870
1	1,6,9	400	2821	768	3589	1,4,9	1423	1870	3293
2	1,6,9	400	3185	1033	4218	1,6	542	3293	3835
3	1,6,9	400	3718	1477	5195	1,6	735	3835	4570
4	1,6,9	450	3973	1671	5644	1,6	434	4570	5004
5	1,6,9,11	500	4197	1817	6014	1,4,6	789	5004	5793
6	1,4,9,11	600	4940	2088	7028	1,6	811	5793	6604
7	1,4,9,11	600	5420	2205	7625	1,6	739	6604	7443
8	1,4,9,11	600	5992	2286	8278	1,6	871	7443	8314
9	1,4,9,10,11	700	6393	2416	8809	1,6	824	8314	9138
10	1,4,9,10,11	750	6802	2445	9247	6,8	739	9138	9877

\+ The existing facilities at locations 6 and 9 are having economic life of 10 years.

Figure 1: Total Cost Comparison of Super-imposed and Optimal Networks at Time t.



(i) The increase in total demand at time t = 5 and 6 is low, which results in the lower total cost at t = 4 and t = 5, to meet the total demand.

(ii) At time t = 6, demand is considered for new demand centers (8,10,16,18) which come into existence at t = 7. The demand centers 4, 14 and 16 are near the facility location 4 and can be supplied at a cheaper cost from this facility instead of being supplied from facility 6. As a result of which a facility is opened at location 4 and facility 6 is dropped from the optimal network. The demand centers 7 and 12 which were earlier being supplied from facility 6 are now supplied from facility 9. The total cost for the network of time t = 7 is higher (a sudden increase at t = 6).

The relative increase in the total cost for the optimal network at t = 5 is lower than at other times.

(b) Facility 6 is very important in the super-imposed network whereas it is important for the optimal network only in the beginning and subsequently it gets dropped. The following explanations can be given:

The super-imposed network never takes into consideration the total demand and its distribution as a whole at any time. At all times, only the increase in the demand is being distributed. As compared to this, the optimal network

takes into consideration the total demand and its distribution as a whole at each decision point.

(c) The analysis proposed in this thesis gives a time $\hat{t}$ at which it is profitable to changeover from the super-imposed to the optimal network, whereas this changeover was not profitable at $\hat{t} - 1$. This $\hat{t}$ is determined based on the cost considerations at a given point only. It is possible that if we take into consideration, some future time and consider the costs upto that time, then this changeover may be earlier than $\hat{t}$. We now give an analysis to determine the most profitable changeover time, assuming the planning horizon to be upto $\hat{t}$.

For the numerical example considered above, $\hat{t} = 8$.

Let us assume that the demand at any demand center j, at time $t < 8$ is met with in the same way as it was met with by the optimal network of $\hat{t} = 8$. The following table gives the total annual costs of building the network at time t $(t = 1,2,\ldots,8)$.

The present worth of cost flow is used as the criterion for determining the most economic period for building the optimal network.

Table 4: Economic Evaluation of Building the Optimal Network at Various Time Periods.

Total Annual cost at t	Optimal Network Built at time t: 1	2	3	4	5	6	7	8
0	1870	1870	1870	1870	1870	1870	1870	1870
1	3791	3293	3293	3293	3293	3293	3293	3293
2	3785	4418	3835	3835	3835	3835	3835	3835
3	4281	4281	5358	4570	4570	4570	4570	4570
4	4509	4509	4509	5730	5004	5004	5004	5004
5	4986	4986	4986	4986	6300	5793	5793	5793
6	5452	5452	5452	5452	5452	6940	6604	6604
7	6032	6032	6032	6032	6032	6032	7637	7443
8	6590	6590	6590	6590	6590	6590	6590	8276
Present worth at period t = 0	33776	33808	34242	34492	34802	35362	36190	36838

The PW values given in Table 4 indicate that if the optimal network $\hat{t}$ is built at time t = 1 or 2, the overall total cost shall be less than the cost of building such a network at a later date. However, it needs to be pointed out that the above analysis gives the worst estimation of the total cost. The optimal network of time t could have been built in stages to cater to the total demand $\sum_j d_{jt}$ at time t

instead of building this network having capacity to supply $\sum_j d_{j\hat{t}}$ at time t, thus resulting in the savings in the investment.

4.4 GENERAL DISCUSSIONS:

In practice, any increase in demand is met with by tinkering with the existing network. The increase in demand is accommodated by the nearby existing facilities. Also if required, the distribution network is changed so as to cater to the total demand economically. Let this network be called evolved network. The total cost of the evolved network is lower than that of the super-imposed network considered in this thesis.

In most of the instances the cost of changeover from the evolved network at time t to the optimal network shall be less than the cost of changing over from the super-imposed network to the optimal network. Thus the total cost of the optimal network corresponding to the evolved network shall be less than the cost of the optimal network. It is envisaged that the difference in total cost between the evolved network and the optimal network (corresponding to the evolved network) shall be more or less same as the difference in the total costs at that time between the super-imposed network and the corresponding optimal network. Thus this type of analysis can be considered approximately

close to the real life situation as far as the cost comparison (difference between the two costs) is concerned.

4.5 SCOPE FOR FURTHER RESEARCH:

There are many facets of the problem attempted in this dissertation, which need further investigations. Efforts need to be made to develop methodologies for considering the following problems:

(a) to show that the nature of the total cost curve for the optimal and the super-imposed network is concave and convex increasing respectively.

(b) to obtain mathematically, the subsequent changeover points once the first changeover point is obtained, assuming that at each such point the changeover is carried over from the super-imposed to the optimal network.

(c) to obtain the subsequent changeover points considering that at each point the optimal network is built ahead of $\hat{t}$ on the basis of the analysis similar to 4.3(c).

(d) to predict the first changeover point knowing the total cost curve of the super-imposed network and a few starting points of the total cost curve for the optimal network, assuming that the total demand pattern is known.

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APPENDIX

```
$IBJOB
$IBFTC MAIN
C*****
C         HEURISTIC ALGORITHM FOR FACILITIES LOCATION
C         M IS THE SET OF LOCATIONS OUT OF WHICH FACILITIES ARE TO BE CHOS
C         N IS THE NOS. OF DEMAND CENTERS
C         D(J) IS THE DEMAND AT DEMAND CENTER J
C         CCN1 AND CCN2 ARE THE CONSTANTS OF OPERATING COST
C         FM IS THE FIXED FACILITY COST
C         DIFF IS THE INCREMENTAL FIXED FACILITY COST
C         OPCOST IS THE VARIABLE OPERATING COST
C         C IS THE TRANSPORTATION COST PER UNIT PER UNIT DISTANCE
C         A(I) IS THE NOS. OF UNITS TO BE PRODUCED AT FACILITY I
C*****
      DIMENSION D(20),Y(20,20),T(20,20),A(20),X(20,20),U(20,20)
     1,FM(4),PZ(11),CF(11),IX(20),RX(20,20)
      READ 701,M,N
  701 FORMAT(2I2)
      READ 702,CCN1,CCN2
  702 FORMAT(2F8.4)
      READ 703,((Y(I,J),J=1,N),I=1,M)
  703 FORMAT(20F4.0)
      READ 704, (FM(I), I=1,3)
  704 FORMAT(3F8.0)
      READ 705,C
  705 FORMAT(F5.1)
      READ 706,(D(J),J=1,N)
  706 FORMAT(20F4.0)
      COST = 99999.
      IDC=1
      CF(1)=0.
      PZ(1)=0.
      ICOUNT = 0
      W = 0.
 1101 P = W
 1100 CONTINUE
      DO 1102 I=1,M
 1102 IX(I)=0
      COSTT = COST
      IF (ICOUNT.GT.0) GO TO 110
C*****
C         T(I,J) IS THE TRANSPORTATION COST MATRIX
C*****
      DO 1 I = 1,M
      DO 1 J= 1,N
    1 T(I,J) = Y(I,J)*C
      PRINT 601
  601 FORMAT(1H1,10X,* DEMAND MATRIX *,//)
      KK = N
      IF (N.GT.10)  KK = 10
```

```
      PRINT 602,(E(J),J=1,KK)
  602 FORMAT(10X,10F10.2)
      IF (KK - N) 603,604,604
  603 KK = KK + 1
      PRINT 605,(E(J),J=KK,N)
  605 FORMAT(10X,10F10.2)
  604 CONTINUE
      PRINT 500
  500 FORMAT(//,10X,* TRANSPORTATION COST MATRIX *)
      JJ = N
      IF (N.GT.10)  JJ = 10
      DO 100 I = 1,M
      PRINT 510,(T(I,J),J=1,JJ)
  510 FORMAT(10X,10F10.2)
  100 CONTINUE
      IF(JJ - N) 31,41,41
   31 PRINT 11
   11 FORMAT(1H0,10X,* TRANSPORTATION MATRIX CONTINUED *)
      JJ = JJ + 1
      DO 700 I = 1,M
      PRINT 800,(T(I,J),J=JJ,N)
  800 FORMAT(10X,10F10.2)
  700 CONTINUE
   41 CONTINUE
C*****
C     INDIFFERENT SOLUTION ( SOLUTION WITH NO FACILITY COST )
C*****
      CALL [illegible]V1(T,D,N,M,[illegible],COST)
      PRINT 1001
 1001 FORMAT(///,10X,* ASSIGNMENT MATRIX *)
      MM = N
      IF(N.GT.10)  MM = 10
      DO 550 I = 1,M
      PRINT 552,(A(I,J),J=1,MM)
  552 FORMAT(10X,10F10.2)
  550 CONTINUE
      IF  (MM - N) 23,14,14
   23 PRINT 24
   24 FORMAT(1H0,10X,* ASSIGNMENT MATRIX CONTINUED *)
      MM = MM + 1
      DO 555 I = 1,M
      PRINT 556,(A(I,J),J=MM,N)
  556 FORMAT(10X,10F10.2)
  555 CONTINUE
   14 CONTINUE
      PRINT 13,COST
   13 FORMAT(1H0,10X,* TOTAL COST FOR INDIFFERENT SOLUTION IS *,F15.8//
C*****
C     SOLUTION FOR TRANSPORTATION COST PLUS FIXED FACILITY COST
C*****
```

```
110 ICOUNT = 0
    ICOUNT = ICOUNT + 1
    DO 2[illegible] I=1,M
    DO [illegible] J=1,N
2[illegible] [illegible](I,J)=[illegible](I,J)
2[illegible] [illegible] = COST
    DO 20 I = 1,M
    DO 20 J = 1,N
    W = 0.000001
    DO [illegible] JI = 1,N
[illegible] W = W + [illegible](I,JI)
    IF(W.LE.0.0000001) F=1.
    IF(W.GT.1..AND.W.LE.[illegible]00.) F=FM(1)*.1*FLOAT(ITDC)
    IF(W.GT.[illegible]00..AND.W.LE.[illegible]00.) F=FM(2)*0.1*FLOAT(ITDC)
    IF(W.GT.[illegible]00..AND.W.LE.1000.) F=FM(3)*0.1*FLOAT(ITDC)
    IF(W.GT.1000..AND.W.LE.1500.) F=FM(4)*0.1*FLOAT(ITDC)
    IF(W.GT.1[illegible]00.) F=FM(5)*0.1*FLOAT(ITCC)
20  U(I,J) = T(I,J) + F/W
    CALL [illegible]VE(U,D,N,M,[illegible],COST)
    IF(COST[illegible]) GO TO 25
    DO [illegible] I = 1,M
    [illegible](I) = 0
    DO [illegible] J = 1,N
[illegible] [illegible](I) = [illegible](I) + [illegible](I,J)
    W = 0.
    [illegible] = M
    DO [illegible]0 I=1,M[illegible]
    IF([illegible](I).GE.1.) W=W+1.
    DO [illegible]00 J = 1,N
    IF([illegible](I).LT.1.) T(I,J) = 99999.
[illegible]00 CONTINUE
    CALL [illegible]VE(T,D,N,M,[illegible],COST)
    DO [illegible]01 I = 1,M
    [illegible](I) = 0
    DO [illegible]01 J = 1,N
[illegible]01 [illegible](I) = [illegible](I) + [illegible](I,J)
C*****
C     [illegible]T-[illegible]   JUSTIFYING EACH FACILITY
C*****
    [illegible]=1
[illegible] CONTINUE
    J=1
    DO 200 I=1,M
    IF([illegible](I).LT.1.) GO TO 200
    IF([illegible](I).GE.[illegible]) GO TO [illegible]
    [illegible]NDL=1
    DO 201 K = 1,M
    IF ([illegible](K).LT.1.) GO TO 201
    IF (K.[illegible].GO.[illegible]) GO TO 201
    IF([illegible]([illegible]DU).[illegible]G.K) GO TO [illegible]
```

```
      [illegible] = [illegible]
      [illegible] 202 J=1,[illegible]
202   [illegible] = [illegible] + ([illegible](I,J) - [illegible](K,J))* [illegible](K,J)
      MY = K
      F=0.
      IF([illegible](K).GT.[illegible].AND.[illegible](K).L[illegible]CO.)    F=FM(1)*0.1*FLOAT(ITDC)
      IF([illegible](K).GT.[illegible]CO.AND.A(K).L[illegible]CO.)    F=FM(2)*0.1*FLOAT(I[illegible]DC)
      IF([illegible](K).GT.[illegible]00.AND.A(K).L[illegible]1COO.)    F=FM([illegible])*0.1*FLOAT([illegible]TDC)
      IF([illegible](K).GT.1CCC.[illegible]AND.A(K).LT.1[illegible]00.) F=FM(4)*0.1*FLOAT(I[illegible]DC)
      IF([illegible](K).GT.1[illegible]CC.) F=FM(5)*0.1*FLOAT(ITDC)
      [illegible]((C[illegible]1*[illegible]([illegible])-CCN2*[illegible](I)*[illegible](I))+C[illegible]1*A(K)-CCN2*A(K)*A(K)-
     1(C[illegible]1*([illegible](I)+A(K)))+(CCN2*(A(I)+A(K))*(A(I)+A(K)))+F .GT [illegible]S)
     [illegible] 20[illegible]
      G[illegible] 201
[illegible] [illegible]DL=[illegible]DL+1
201   [illegible]
      [illegible] 20[illegible]
[illegible] [illegible]=[illegible]+1
200   [illegible]
      [illegible] 20[illegible]
20[illegible] [illegible] 20[illegible] J =1,N
20[illegible] [illegible](MY,J) = 99999.
      [illegible]([illegible])=MY
      [illegible]=[illegible]+1
      [illegible]=[illegible]-1
      CALL [illegible]V[illegible] ([illegible],D,[illegible],M,[illegible],COST)
      DO 21[illegible] [illegible]=1,[illegible]
      [illegible]([illegible])=0.
      [illegible] 21[illegible] J=1,[illegible]
214   [illegible]([illegible])=[illegible]([illegible])+[illegible]([illegible],J)
      [illegible]
20[illegible] [illegible]
      [illegible]F[illegible]K=0.
      [illegible]CK=0.
      DO [illegible] K=1,[illegible]
      F=0.
      [illegible]FF=0.
      IF([illegible](K).GT.[illegible].AND.[illegible](K).L[illegible]CO.)    DIFF=FM(1)*0.1
      IF([illegible](K).GT.[illegible]CO.AND.A(K).L[illegible]600.)    DIFF=FM(2)*0.1
      IF([illegible](K).GT.[illegible]00.AND.A(K).L[illegible]1COO.)   DIFF=FM([illegible])*0.1
      IF([illegible](K).GT.1COO..AND.A(K).LT.1[illegible]0.) DIFF=FM(4)*0.1
      IF([illegible](K).GT.1[illegible]0.) D[illegible]FF=FM([illegible])*0.1
      IF([illegible](K).GT.[illegible].AND.[illegible](K).L[illegible]CO.)    F=FM(1)*0.1*FLOAT(ITDC)
      IF([illegible](K).GT.[illegible]CC.AND.A(K).L[illegible]CO.)    F=FM(2)*0.1*FLOAT(I[illegible]DC)
      IF([illegible](K).GT.[illegible]CO.AND.A(K).L[illegible]1COO.)    F=FM([illegible])*0.1*FLOAT([illegible]TDC)
      IF([illegible](K).GT.1[illegible]CO.[illegible]AND.[illegible](K).LT.1[illegible]00.) F=FM(4)*0.1*FLOAT(I[illegible]DC)
      IF([illegible](K).GT.1[illegible]CO.) F=FM(5)*0.1*FLOAT(ITDC)
      CH[illegible]CK=CH[illegible]CK+DIFF
      [illegible]CK=[illegible]CK+F
[illegible] CONTINU[illegible]
```

```
      CH(ITDC+1)=CHICK
      PI(ITDC+1)=PICK
      DO 200 IK =1,M
  200 COST = COST + (CON1*A(IK)) - (CON2*A(IK)*A(IK))
C*****
C     ENSURING NO PROFITABLE FACILITY IS ELIMINATED
C*****
      IF(COST.GT.XCOST+CH(ITDC).AND.ITDC.EQ.10) PRINT 2002,P
      IF(COST.LT.XCOST+CH(ITDC).AND.ITDC.EQ.10) PRINT 2002,W
      IF(ITDC.LT.10) GO TO 800
      PRINT 1, PI(ITDC+1)
  800 CONTINUE
      IF(COST.LT.XCOST+CH(ITDC)) GO TO 2000
      DO 222 I=1,N
      DO 222 J=1,M
  222 X(I,J)=PX(I,J)
      COST=XCOST+PI(ITDC)
      GO TO 2000
 1000 COST=COST+PI(ITDC+1)
 2000 CONTINUE
      IF(ITDC.LT.10) GO TO 818
      PRINT 111,COST
  111 FORMAT(1H0,10X,* OPTIMAL SOLUTION IS *,F10.2)
      PRINT 660
  660 FORMAT(///,10X,* THE ASSIGNMENT MATRIX FOR THE ABOVE SOLUTION *)
      KK = N
      IF (N.GT.10)  KK = 10
      DO 661 I=1,M
      PRINT 662,(X(I,J),J=1,KK)
  662 FORMAT(10X,10F10.2)
  661 CONTINUE
      IF(KK - N)  3,1,1
    3 PRINT 664
  664 FORMAT(///,10X,* ASSIGNMENT MATRIX CONTINUED *)
      KK = KK + 1
      DO 665 I = 1,M
      PRINT 667,(X(I,J),J=KK,N)
  667 FORMAT(10X,10F10.2)
  665 CONTINUE
    1 CONTINUE
  818 CONTINUE
      ITDC=ITDC+1
    1 FORMAT(1H0,10X,* FIXED FACILITY COST = *,2X,F10.0)
 2002 FORMAT(1H1,10X,* NUMBER OF FACILITIES = *,2X,F4.0)
      IF(ITDC.LT.10.AND.COST.LT.XCOST+CH(ITDC-1)) GO TO 1101
      IF(ITDC.LT.10) GO TO 1100
 1000 CONTINUE
      STOP
      END
```

```
$IBFTC SUB1
C*****
C     THE FOLLOWING SUBROUTINE SAV1  MINIMISES THE TOTAL  TRANSP.  COST
C*****
      SUBROUTINE SAV1(T,D,N,M,X,COST)
      DIMENSION T(20,20),D(20),X(20,20)
      DO 5 I=1,N
      DO 5 J=1,M
    5 X(I,J) = 0.
      COST = 0.
      DO 20 J= 1,M
      AMIN = T(1,J)
      IMIN = 1
      DO 10 I = 2,N
      IF(AMIN.LT.T(I,J))  GO TO 10
      AMIN=T(I,J)
      IMIN = I
   10 CONTINUE
      X(IMIN,J) = X(IMIN,J) + D(J)
      COST = COST + AMIN * D(J)
   20 CONTINUE
      RETURN
      END
$ENTRY
```

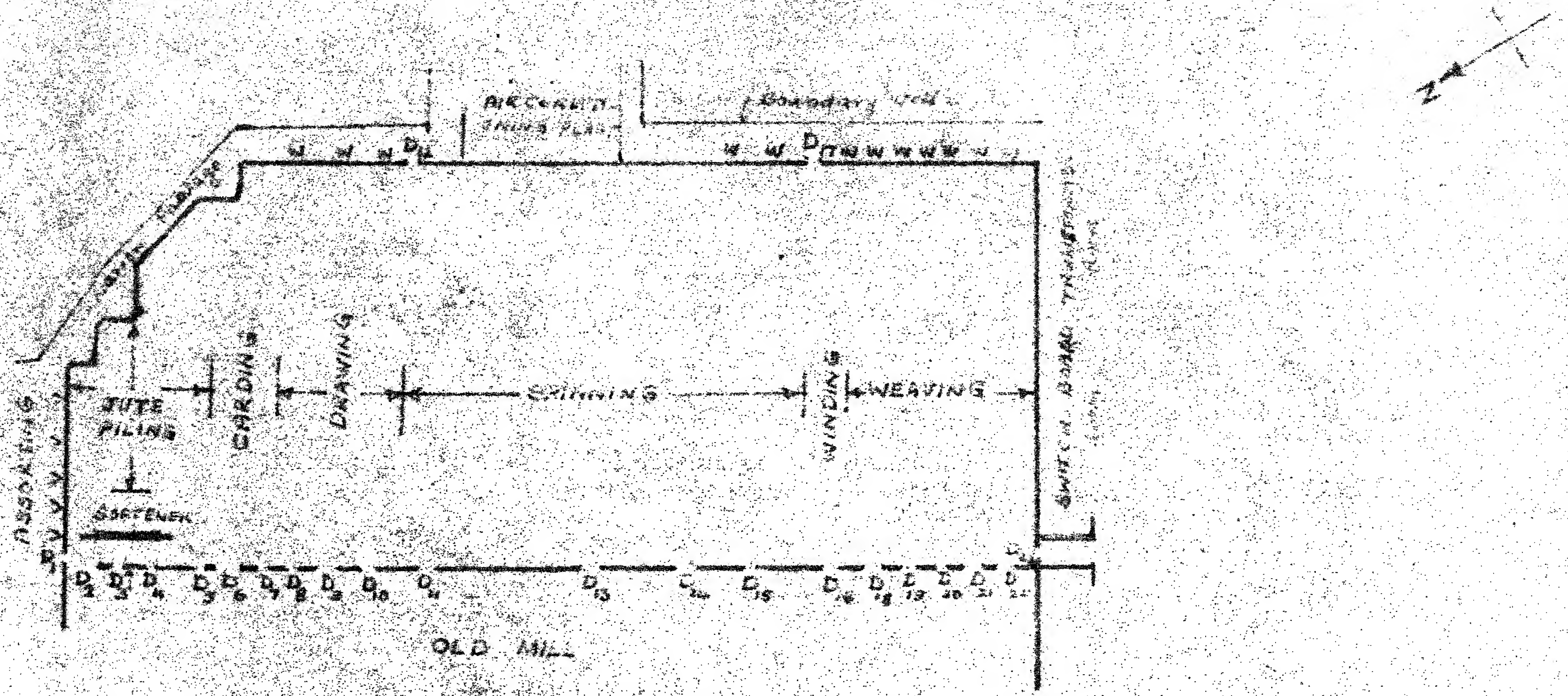

Fig 5.2. GENERAL LAY-OUT
OF FACTORY ROOM [JUTE MILLS]
[NOT TO SCALE]

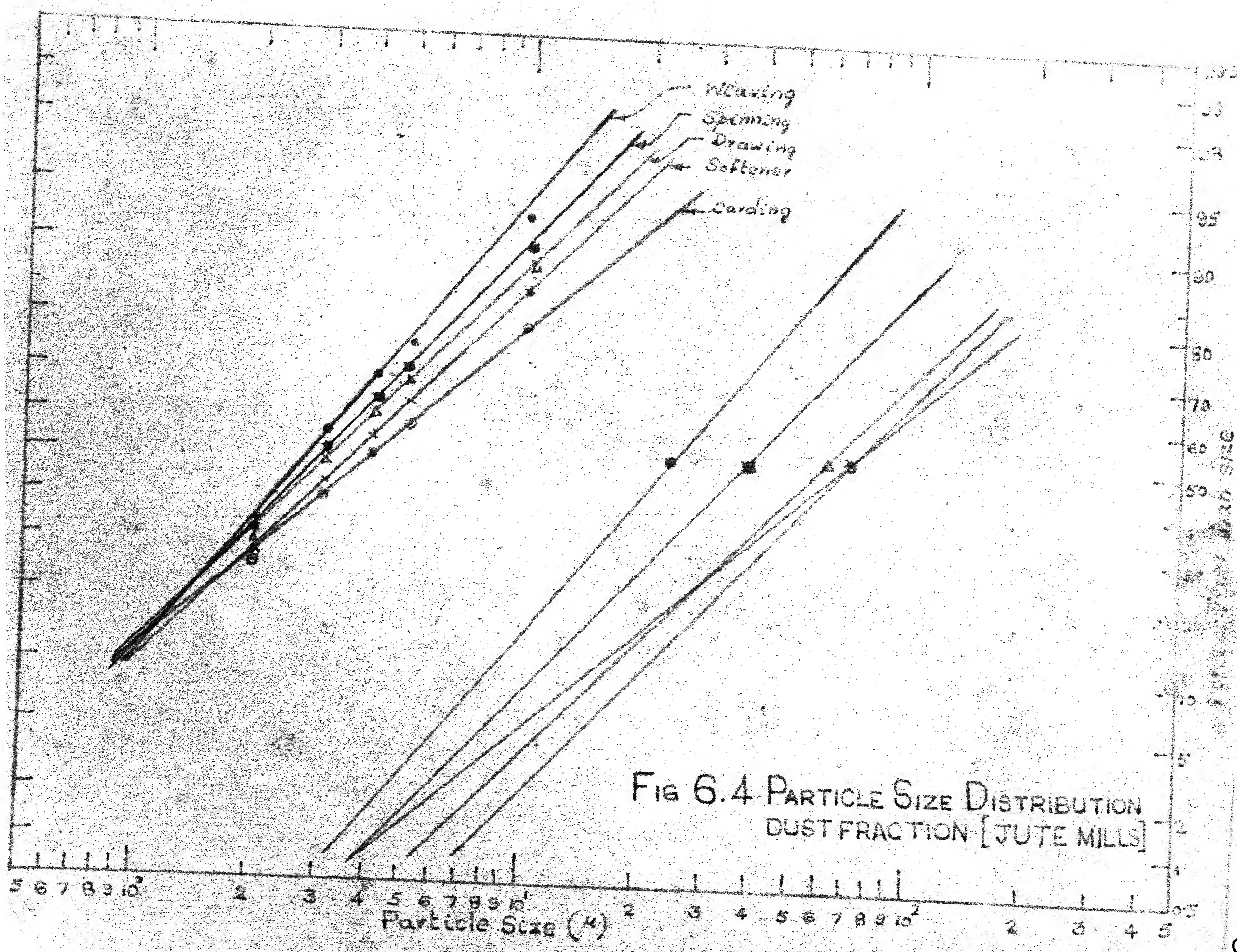

Fig 6.4 Particle Size Distribution
Dust Fraction [Jute Mills]

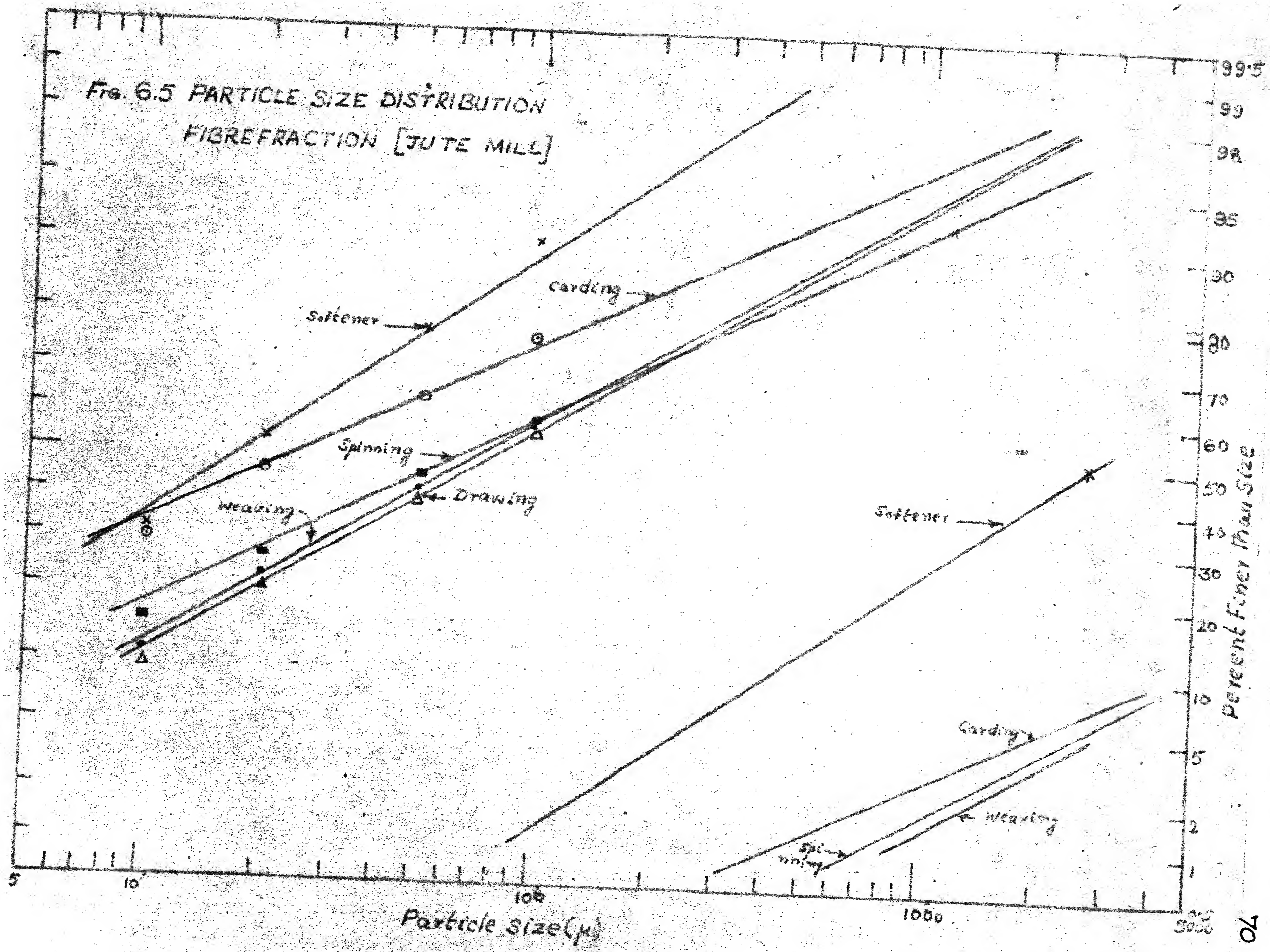

FIG. 6.5 PARTICLE SIZE DISTRIBUTION
FIBREFRACTION [JUTE MILL]

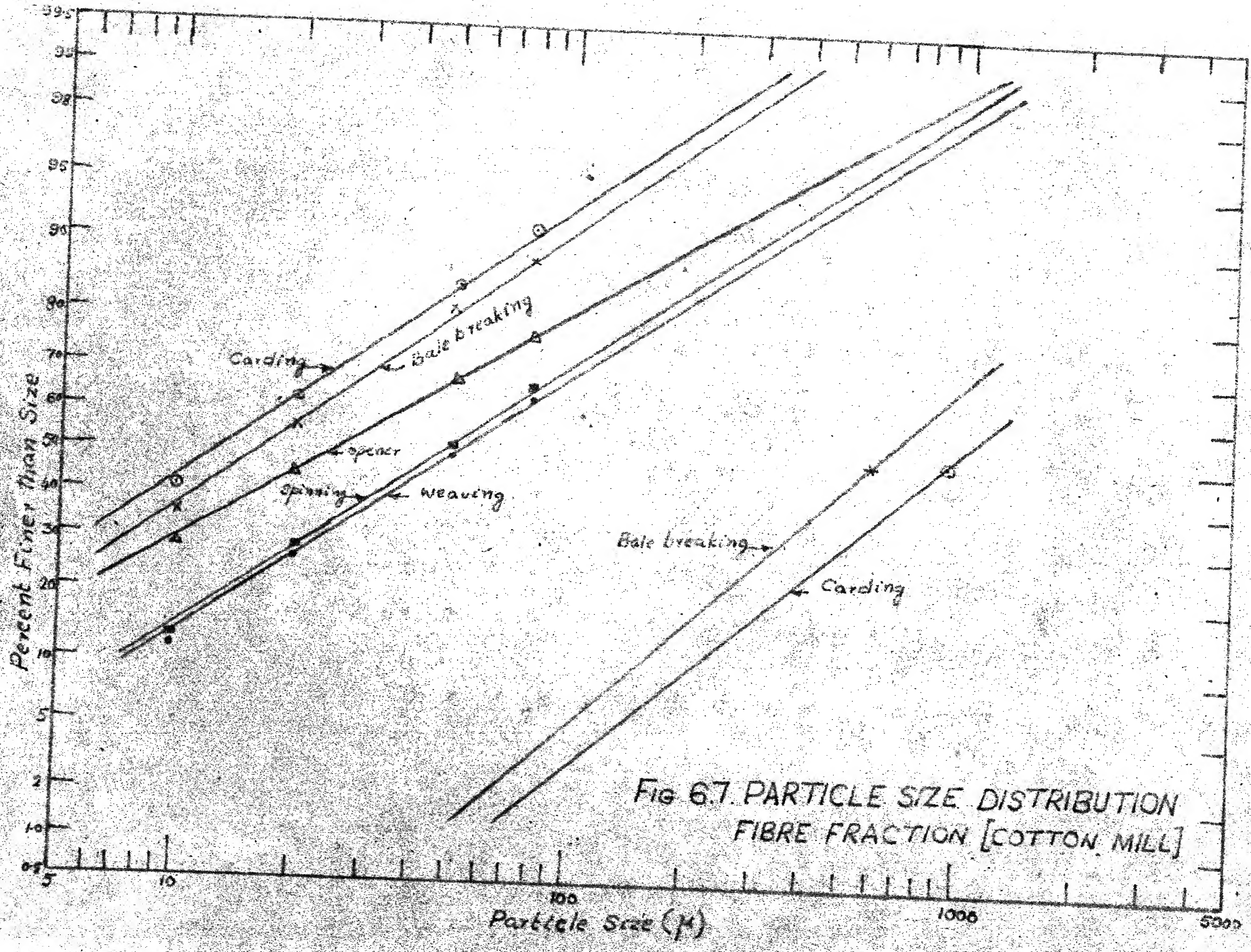

FIG 6.7 PARTICLE SIZE DISTRIBUTION
FIBRE FRACTION [COTTON MILL]

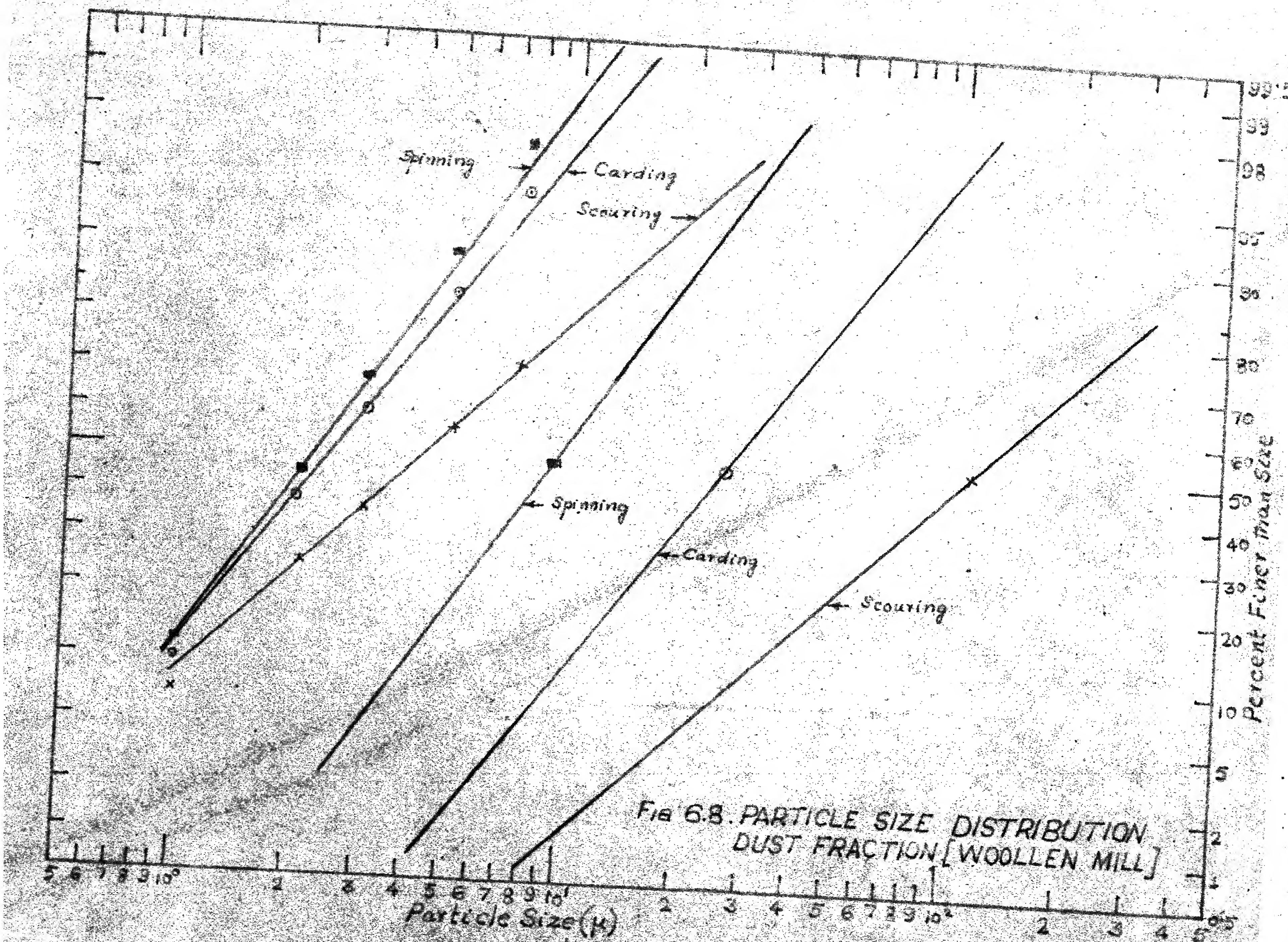

Fig 6.8. PARTICLE SIZE DISTRIBUTION DUST FRACTION [WOOLLEN MILL]

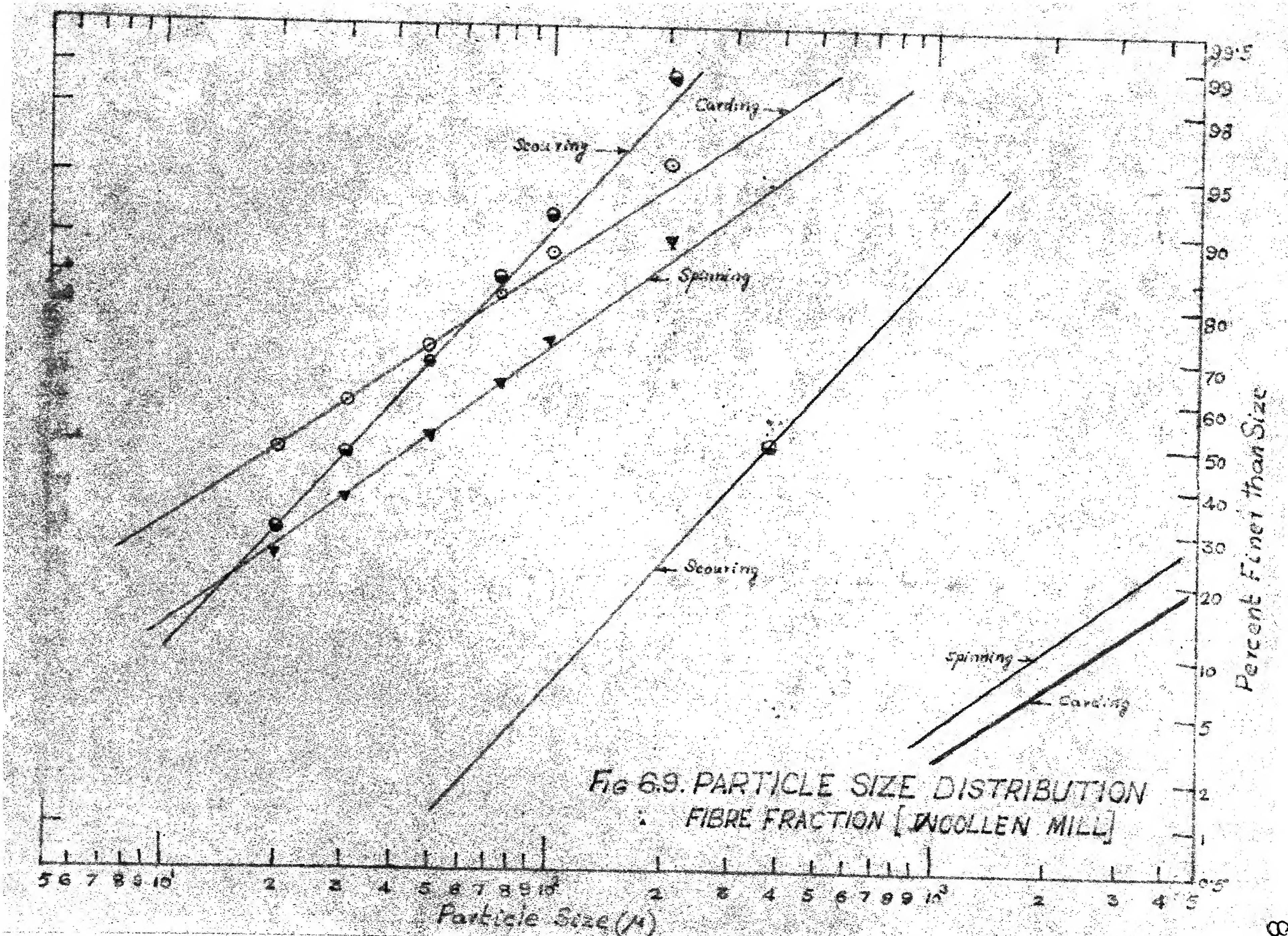

Fig 6.9. PARTICLE SIZE DISTRIBUTION
: FIBRE FRACTION [WOOLLEN MILL]